All Order Loop Renormalizable Wess-Zumino Model on Bosonic-Fermionic Noncommutative Superspace

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Abstract

We extend the ordinary Wess-Zumino model to the Bosonic-Fermionic noncommutative (BFNC) superspace and study its quantum property. In our previous work, we have proved that one BFNC Wess-Zumino model is one-loop renormalizable up to the second order of BFNC parameters. Inspired by this, in the present paper we construct a new formulation of BFNC Wess-Zumino models, analyze its all possible effective actions by using the dimensional analysis method, and further prove that the new BFNC Wess-Zumino model is renormalizable to all order loop expansion.

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Contents

1 Introduction

By deforming the ordinary spacetime and superspace to a noncommutative (NC) spacetime and non-anticommutative (NAC) superspace, respectively, and then constructing physical models on the NC spacetime and NAC superspace, we can acquire a deeper understanding of quantum field theory. One of the most interesting properties is the renormalization of quantum field theory on the NC spacetime and NAC superspace. For instance, in ref. [1] on the one hand the simplest NAC superspace was given and the possibility to construct the Bosonic-Fermionic noncommutative (BFNC) superspace was pointed out, and on the other hand the Wess-Zumino model and the super Yang-Mills model were generalized to the NAC superspace and the effect of NAC deformation on the physical models was investigated. In addition, in the later researches [2–6] the NAC Wess-Zumino model was proved to be renormalizable to all order loop expansion.

From the point of view of string theory [7–10], the NAC superspace can be obtained under certain conditions and furthermore the BFNC superspace [11] can be predicted where the Bosonic coordinates no longer commute with the Fermionic coordinates.

Compared with a lot of studies on the models based on the NAC superspace, the investigations of models defined on the BFNC superspace are few. The main reason is that the BFNC $\star$-product has to be expanded up to the fourth order in BFNC parameters, which results in a quite complex action with a large number of terms. In our recent work [12] we generalize the ordinary Wess-Zumino model to the BFNC superspace. In terms of the iterative method, we construct the deformed model and prove that it is one-loop renormalizable up to the second order of the BFNC parameters.

In this paper we construct a new formulation of BFNC Wess-Zumino models and explore its renormalization property to all order loop expansion. The analysis will be carried out in superspace and the method we shall use is similar to that of ref. [5]. We adopt the same notations as in ref. [5], such as the conservation of mass dimension, the $U(1)_R$ R-symmetry, and the $U(1)_\Phi$ flavor symmetry. Our process of investigation is briefly described below. At first defining the dimension and charge for interacting constants and operators, we can obtain the possible forms of operators in effective actions. Then, using the 1/2 supersymmetry we can remove some operators. After constructing all possible forms of BFNC parameters, we can separate the remaining operators into two classes, one class is covered by the deformed model and the other one can be obtained in higher order loop expansion. Finally, we can prove that the generalized Wess-Zumino model on the BFNC superspace is renormalizable to all order loop expansion up to the second order of BFNC parameters.

The paper is organized as follows. In section 2 we give a new formulation of BFNC Wess-Zumino

models. In section 3 we establish the framework to analyze all possible forms of operators in effective actions. We list all the possible operators in section 4 and then compare it with the Wess-Zumino action in section 5. We analyze the contribution from higher order loop expansion in section 6. At last, we present our conclusion and outlook in section 7.

2 New Wess-Zumino Model on BFNC Superspace

In order to study the renormalization to all order loop expansion by using symmetry analysis method, we give a new Wess-Zumino model on the BFNC superspace,

$$\begin{aligned} \mathcal{S}'_{\rm NC} &= \int d^4x \Big\{ \Phi^+ \star \Phi|_{\theta^2\bar{\theta}^2} + \frac{m}{2}\Phi \star \Phi|_{\theta^2} + \frac{g}{3}\Phi \star \Phi \star \Phi|_{\theta^2} \\ &\qquad + \frac{m^*}{2}\Phi^+ \star \Phi^+|_{\bar{\theta}^2} + \frac{g^*}{3}\Phi^+ \star \Phi^+ \star \Phi^+|_{\bar{\theta}^2} \Big\}, \end{aligned} \tag{1}$$

where $\star$-product, Φ, and Φ^+ are same as that in action $\mathcal{S}_{\rm NC}$ [12]. Here we only introduce new parameters m^* and g^* which are complex conjugates of m and g, respectively.

3 Symmetry Analysis

In this section we establish the framework for symmetry analysis.

In ref. [2] the global symmetries, such as the $U(1)_R$ R-symmetry and $U(1)_\Phi$ flavor symmetry, were found in the Wess-Zumino model on the NAC superspace. The global symmetries can be defined by using charge of symmetry. In Table 1, we define the mass dimension d, $U(1)_R$ R-symmetry charge R, and $U(1)_\Phi$ flavor symmetry charge S for the constants and operators. We note that we have also defined the charge for the BFNC parameter ${\Lambda^k}_\alpha$.

The effective action can be written in the following form [5],

$$\Gamma = \int d^4x \; \lambda \; \mathcal{O}, \tag{2}$$

where the constant is defined as,

$$\lambda \sim \Lambda_{UV}^d g^{x-R} g^{*x} \left(\frac{m}{\Lambda_{UV}}\right)^y \left(\frac{m^*}{\Lambda_{UV}}\right)^{y+\frac{S-3R}{2}} {\lambda_i}^{\omega_i} \tag{3}$$

where m and m^* are masses, g and g^* are interacting constants, Λ_{UV} is ultraviolet cutoff, λ_i is parameter without dimension, and x, y, and ω_i are non-negative integers. By construction for λ the mass dimension is d, the $U(1)_R$ R-symmetry charge is R, the $U(1)_\Phi$ flavor symmetry charge is S.

	dim	$U(1)_R$	$U(1)_S$		dim	$U(1)_R$	$U(1)_S$
m	1	0	-2	m^*	1	0	2
g	0	-1	-3	g^*	0	1	3
$\left(\Lambda^k{}_\alpha\right)^2$	-3	2	0	V	-5	2	0
$d^4\theta$	2	0	0	θ^4	-2	0	0
Φ	1	1	1	Φ^+	1	-1	-1
D_α	$\frac{1}{2}$	-1	0	$\bar{D}_{\dot{\alpha}}$	$\frac{1}{2}$	1	0
D^2	1	-2	0	$\bar{D}^2$	1	2	0
∂_k	1	0	0	$\Box$	2	0	0

Table 1: Mass dimension and charge for constants and operators.

We note that in our previous work [12] the action contains only m and g. We can modify it to have the right mass dimension and neutral charge by redefining the parameters of invariant subsets. The resulted action contains m, m^*, g, and g^*.

The general form of operators in effective actions is

$$\mathcal{O} = d^4\theta \left(D^2\right)^\gamma \left(\bar{D}^2\right)^\delta \left(D\sigma^k\partial_k\bar{D}\right)^\eta \Box^\zeta V^\rho \Phi^\alpha \left(\Phi^+\right)^\beta , \tag{4}$$

where γ, δ, η, ζ, ρ, α, and β are non-negative integers, k is spacetime index. $\Box$ represents $\partial_k\partial_l$, which is not only combining with η^{kl}, but also with any constant forms with indices k and l. We note that this is different from the case on the NAC superspace [5]. We have defined $V \equiv \left(\Lambda^k{}_\alpha\right)^2 \theta^4$.

We note that eq. (4) does not contain spacetime or spinor indices. The indices of $\left(\Lambda^k{}_\alpha\right)^2$ will combine with indices of D_α and ∂_k.

By using Table 1 we can calculate the mass dimension of $\mathcal{O}$,

$$2 + \alpha + \beta + \gamma + \delta + 2\zeta + 2\eta - 5\rho, \tag{5}$$

$U(1)_R$ R-symmetry charge of $\mathcal{O}$,

$$\alpha - \beta + 2(-\gamma + \delta + \rho), \tag{6}$$

and $U(1)_\Phi$ flavor symmetry charge of $\mathcal{O}$,

$$\alpha - \beta. \tag{7}$$

By summing the symmetry charge of constant eq. (3) and operator eq. (4) and by using the constraint that the mass dimension in the integral $\int d^4x$ is 4 and the constraints that $U(1)_R$ and $U(1)_\Phi$ charges

of the effective action are 0, we obtain

$$\begin{aligned} d &= 2-\alpha-\beta-\gamma-\delta-2\zeta-2\eta+5\rho, \\ R &= -\alpha+\beta+2\gamma-2\delta-2\rho, \\ S &= -\alpha+\beta. \end{aligned} \tag{8}$$

By using eq. (3) we can calculate the power of Λ_{UV},

$$P = d + \frac{3R}{2} - \frac{S}{2} - 2y. \tag{9}$$

After replacing d, R, S in eq. (9) by eq. (8) we obtain

$$P = 2 - 2y - 2\alpha + 2\gamma - 4\delta - 2\zeta - 2\eta + 2\rho. \tag{10}$$

As $\lambda \propto {\Lambda_{UV}}^P$ and Λ_{UV} becomes infinity, the effective action eq. (2) is divergent only when $P \geq 0$, so the divergent parts of the effective action correspond to

$$P \geq 0. \tag{11}$$

When acting on superfield, $D\partial D$ will use a pair of Φ and Φ^+, and there is only one $D\partial D$ acting on a pair of Φ and Φ^+, so the number of Φ that allows D^2 acting on is $\alpha - \eta$, and the number of Φ^+ that allows $\bar{D}^2$ acting on is $\beta - \eta$. So we have

$$\gamma \leq \alpha - \eta, \quad \delta \leq \beta - \eta. \tag{12}$$

All of the parameters represent the number of operators, they are non-negative integers,

$$\begin{aligned} &\gamma \geq 0, \qquad \delta \geq 0, \qquad \alpha \geq 0, \qquad \eta \geq 0, \qquad \rho \geq 0, \\ &\zeta \geq 0, \qquad x \geq 0, \qquad y \geq 0, \qquad \beta \geq 0. \end{aligned} \tag{13}$$

Approximation to second order of BFNC parameters corresponds to

$$\rho = 1. \tag{14}$$

The powers of g, g^*, m, and m^* in eq. (3) are non-negative,

$$\begin{aligned} &x + \alpha - \beta - 2\gamma + 2\delta + 2\rho \geq 0, \\ &y + \alpha - \beta - 3\gamma + 3\delta + 3\rho \geq 0, \\ &y \geq 0, \quad x \geq 0. \end{aligned} \tag{15}$$

4 Result by Symmetry Analysis

In this section, we give all operator forms and separate them into four classes.

By solving the constraints eq. (10), (11), (12), (13), (14), and (15), we obtain the solutions, then substitute them in the definition eq. (4), we obtain all operator forms.

Their total number is finite. This means that the model is renormalizable. We note from the solution that we can only know the number of derivatives and fields in the operators, we do not know their exact combination.

In the following we will compare them with the one-loop renormalizable action $S_{(3)}$ obtained in ref. [12], we will determine if they are contained in that action.

4.1 First Class

Operators in eq. (16),

$$\begin{aligned}
&\Phi, \quad \Phi\Phi, \quad \Phi\Phi^+, \quad \Phi\Phi^+\Phi^+, \quad \Phi\Phi^+\Phi^+\Phi^+, \\
&\Phi\Phi^+\Phi^+\Phi^+\Phi^+, \quad \Phi\Phi^+\Phi^+\Phi^+\Phi^+\Phi^+, \quad \Phi\Phi\Phi^+, \\
&\Phi\Phi\Phi^+\Phi^+, \quad \Phi\Phi\Phi^+\Phi^+\Phi^+, \quad \Phi\Phi\Phi^+\Phi^+\Phi^+\Phi^+, \\
&\Phi\Phi\Phi^+\Phi^+\Phi^+\Phi^+\Phi^+
\end{aligned} \tag{16}$$

can not be used to construct action satisfying 1/2 supersymmetry invariance. By using the constraints that the effective action is invariant under 1/2 supersymmetry transformation we can remove them.

4.2 Second Class

The operators in eq. (17),

$$\Box\Phi, \quad \Box\Phi^+, \quad \Box\Box\Phi^+, \quad D^2\Box\Phi, \quad D^2\Box\Box\Phi \tag{17}$$

are total derivatives in spacetime integral $\int d^4x$, so we will not consider them.

4.3 Third Class

Operators in eq. (18),

$$\begin{aligned}
&\Phi^+, \quad \Phi^+\Phi^+, \quad \Phi^+\Phi^+\Phi^+, \quad \Phi^+\Phi^+\Phi^+\Phi^+, \\
&\Phi^+\Phi^+\Phi^+\Phi^+\Phi^+, \quad \bar{D}^2\Phi^+, \quad \bar{D}^2\Phi^+\Phi^+,
\end{aligned}$$

$$\bar{D}^2\Phi^+\Phi^+\Phi^+, \quad \bar{D}^2\Phi^+\Phi^+\Phi^+\Phi^+,$$
$$\bar{D}^2\Phi^+\Phi^+\Phi^+\Phi^+\Phi^+, \quad \bar{D}^2\Phi^+\Phi^+\Phi^+\Phi^+\Phi^+\Phi^+ \tag{18}$$

are not contained in the one loop renormalizable action $S_{(3)}$ of Ref. [12]. But we can construct finite 1/2 supersymmetry invariant form by using them. In the following we will prove that they can be produced at higher order loop expansion.

4.4 Fourth Class

From $S_{(3)}$ of Ref. [12], we can find terms which have the following operator forms. We classify them by number of fields.

1 point function,

$$D^2\Phi \tag{19}$$

2 point function,

$$\begin{aligned}
&D^2\Phi\Phi, \quad D^2D^2\Phi\Phi, \quad \Box\Phi^+\Phi^+, \quad \Box\Box\Phi^+\Phi^+,\\
&D^2\Box\Phi\Phi, \quad D^2\Phi\Phi^+, \quad D^2D^2\Box\Phi\Phi, \quad \Box\Phi\Phi^+,\\
&\partial D\overline{D}\Phi\Phi^+, \quad D^2\Box\Phi\Phi^+, \quad D^2\Box\Box\Phi\Phi^+,\\
&D^2\overline{D}^2\Phi\Phi^+
\end{aligned} \tag{20}$$

3 point function,

$$\begin{aligned}
&D^2\Phi\Phi\Phi, \quad D^2D^2\Phi\Phi\Phi, \quad \Box\Phi^+\Phi^+\Phi^+,\\
&\Box\Box\Phi^+\Phi^+\Phi^+, \quad D^2\Phi\Phi^+\Phi^+, \quad D^2\Phi\Phi\Phi^+,\\
&D^2D^2\Box\Phi\Phi\Phi, \quad D^2D^2\Phi\Phi\Phi^+, \quad \Box\Phi\Phi^+\Phi^+,\\
&\partial D\overline{D}\Phi\Phi^+\Phi^+, \quad D^2\Box\Phi\Phi^+\Phi^+, \quad D^2\Box\Phi\Phi\Phi^+,\\
&D^2\partial D\overline{D}\Phi\Phi\Phi^+, \quad D^2\overline{D}^2\Phi\Phi^+\Phi^+,\\
&D^2D^2\overline{D}^2\Phi\Phi\Phi^+
\end{aligned} \tag{21}$$

4 point function,

$$\begin{aligned}
&D^2D^2\Phi\Phi\Phi\Phi, \quad \Box\Phi^+\Phi^+\Phi^+\Phi^+, \quad D^2\Phi\Phi^+\Phi^+\Phi^+,\\
&D^2\Phi\Phi\Phi^+\Phi^+, \quad D^2\Phi\Phi\Phi\Phi^+, \quad D^2D^2\Phi\Phi\Phi\Phi^+,\\
&\Box\Phi\Phi^+\Phi^+\Phi^+, \quad \partial D\overline{D}\Phi\Phi^+\Phi^+\Phi^+,
\end{aligned}$$

$$D^2\Box\Phi\Phi\Phi^+\Phi^+, \quad D^2\partial D\overline{D}\Phi\Phi\Phi^+\Phi^+,$$
$$D^2\overline{D}^2\Phi\Phi^+\Phi^+\Phi^+, \quad D^2D^2\overline{D}^2\Phi\Phi\Phi^+\Phi^+ \tag{22}$$

5 point function,

$$D^2\Phi\Phi\Phi^+\Phi^+\Phi^+, \quad D^2\Phi\Phi\Phi\Phi^+\Phi^+,$$
$$D^2D^2\Phi\Phi\Phi\Phi\Phi^+, \quad \Box\Phi\Phi^+\Phi^+\Phi^+\Phi^+$$
$$\partial D\overline{D}\Phi\Phi^+\Phi^+\Phi^+\Phi^+, \quad D^2\overline{D}^2\Phi\Phi^+\Phi^+\Phi^+\Phi^+ \tag{23}$$

6 point function,

$$D^2\Phi\Phi\Phi\Phi^+\Phi^+\Phi^+ \tag{24}$$

where $\partial \equiv \sigma^k\partial_k$.

5 Constructing Action from Operators

In this section, we will prove that the action constructed from the operators can be classified as two kinds, the first one is contained in $S_{(3)}$, the second one is produced at higher order loop expansion.

5.1 Constant Form

By using eq. (25)

$$\epsilon_{\alpha\beta}, \epsilon^{\alpha\beta}, \eta_{kl}, \eta^{kl}, \left(\sigma^{kl}\right)^{\alpha\beta}, \left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}, \left(\Lambda^k\right)_\alpha \tag{25}$$

we can construct following constant forms, classify them by the number of $k, l, ...$ and $\alpha, \beta, ...$, we have constant form without any indices,

$$\Lambda^2, \quad \sigma\Lambda\Lambda \tag{26}$$

with indices k, l,

$$\left(\eta\sigma\Lambda\Lambda^k\right)^l, \quad \Lambda^{kl}, \quad \Lambda^2\eta^{kl}, \quad \sigma\Lambda\Lambda\eta^{kl} \tag{27}$$

with indices α, β,

$$\Lambda^2\epsilon^{\alpha\beta}, \quad \sigma\Lambda\Lambda\epsilon^{\alpha\beta} \tag{28}$$

with indices $\dot{\alpha}, \dot{\beta}$

$$\Lambda^2\epsilon^{\dot{\alpha}\dot{\beta}}, \quad \sigma\Lambda\Lambda\epsilon^{\dot{\alpha}\dot{\beta}} \tag{29}$$

with indices $k, \dot{\alpha}, \beta$,

$$
\begin{aligned}
&\Lambda^2 \left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}, \quad \sigma\Lambda\Lambda \left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}, \quad \Lambda^{kl}\eta_{ln} \left(\bar{\sigma}^n\right)^{\dot{\alpha}\beta}, \\
&\eta_{nl} \left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta} \left(\eta\sigma\Lambda\Lambda^n\right)^k, \quad \eta_{nl} \left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta} \left(\eta\sigma\Lambda\Lambda^k\right)^n
\end{aligned}
\tag{30}
$$

with indices k, l, n, o,

$$
\begin{aligned}
&\Lambda^2\eta^{kl}\eta^{no}, \quad \sigma\Lambda\Lambda\eta^{kl}\eta^{no}, \quad \eta^{kl}\Lambda^{no}, \\
&\left(\sigma\Lambda\Lambda^{kl}\right)^{no}, \quad \eta^{kl} \left(\eta\sigma\Lambda\Lambda^n\right)^o
\end{aligned}
\tag{31}
$$

with indices k, l, α, β,

$$
\begin{aligned}
&\epsilon^{\alpha\beta} \left(\eta\sigma\Lambda\Lambda^k\right)^l, \quad \epsilon^{\alpha\beta}\Lambda^{kl}, \quad \Lambda^2\eta^{kl}\epsilon^{\alpha\beta}, \quad \sigma\Lambda\Lambda\eta^{kl}\epsilon^{\alpha\beta}, \\
&\Lambda^2 \left(\sigma^{kl}\right)^{\alpha\beta}, \quad \sigma\Lambda\Lambda \left(\sigma^{kl}\right)^{\alpha\beta}, \quad \Lambda^{ko}\eta_{on} \left(\sigma^{nl}\right)^{\alpha\beta}, \\
&\epsilon^{\alpha\zeta}\epsilon^{\beta\iota}\epsilon^{klno}\eta_{np}\eta_{oq}\Lambda^p{}_\zeta\Lambda^q{}_\iota, \quad \epsilon^{\alpha\zeta}\eta_{no} \left(\sigma\Lambda^{ok}\right)^{l\beta} \Lambda^n{}_\zeta, \\
&\epsilon^{\alpha\zeta} \left(\eta\sigma\Lambda^k\right)^\beta \Lambda^l{}_\zeta, \quad \epsilon^{\alpha\zeta}\epsilon^{\beta\iota}\Lambda^k{}_\zeta\Lambda^l{}_\iota, \\
&\eta^{kl}\epsilon^{\alpha\zeta}\eta_{no} \left(\eta\sigma\Lambda^n\right)^\beta \Lambda^o{}_\zeta
\end{aligned}
\tag{32}
$$

We have defined new symbols:

$$
\begin{aligned}
\Lambda^{kl} &\equiv \epsilon^{\alpha\beta}\Lambda^k{}_\beta\Lambda^l{}_\alpha \\
\Lambda^2 &\equiv \eta_{kl}\Lambda^{kl} \\
\sigma\Lambda\Lambda &\equiv \eta_{kn}\eta_{lo} \left(\sigma^{kl}\right)^{\alpha\beta} \Lambda^n{}_\alpha\Lambda^o{}_\beta \\
\left(\sigma\Lambda^{kl}\right)^{n\alpha} &\equiv \left(\sigma^{kl}\right)^{\beta\alpha} \Lambda^n{}_\beta \\
\left(\eta\sigma\Lambda^k\right)^\alpha &\equiv \eta_{ln} \left(\sigma^{nk}\right)^{\beta\alpha} \Lambda^l{}_\beta \\
\left(\eta\sigma\Lambda\Lambda^k\right)^l &\equiv \eta_{no} \left(\sigma^{ok}\right)^{\alpha\beta} \Lambda^n{}_\alpha\Lambda^l{}_\beta \\
\left(\sigma\Lambda\Lambda^{kl}\right)^{no} &\equiv \left(\sigma^{kl}\right)^{\alpha\beta} \Lambda^n{}_\alpha\Lambda^o{}_\beta
\end{aligned}
\tag{33}
$$

5.2 Comparison of Operators and Action

We obtain the allowed operators by using symmetry analysis method. To construct action from them, we have to combine the constant form eq. (26), (27), (28), (29), (30), (31), (32) with them.

In this section we consider the possible combination of constant forms and operators, then compare the result with $S_{(3)}$. We shall observe that some action forms that are not contained in the $S_{(3)}$. In next section we will investigate if they can be produced at high order loop expansion.

To introduce the fact that we should consider when constructing the action, we will use the two point function as example.

They will help us to understand the possible form of action which can be constructed by the operators.

5.2.1 Partial Derivatives

To construct action from operators, one of the difficulty is the action of derivatives on the superfields. For 2 point function, we can use the partial derivatives identity,

$$\int d^4x \left(\partial_k A\right) B = -\int d^4x A \left(\partial_k B\right) \tag{34}$$

to move all the Bosonic derivatives to one of the two fields.

For 2 point function, if there 2 or 4 Bosonic derivatives $\partial_k\partial_l$, $\partial_k\partial_l\partial_n\partial_o$, the constant form that can be combined with them should not be antisymmetry with respect to any two of the indices.

For 3 or higher point function, we can also find a method to arrange the Bosonic derivatives. As a example, for the operators form $\Phi\Phi\Phi^+$, we can demand that there are no Bosonic derivatives on Φ^+, to achieve the goal, we can move all of the Bosonic derivatives acting on Φ^+ to the other two Φ's by using eq. (34).

5.2.2 Distribution of Covariant Derivatives

For acting $D, \bar{D}$ on (anti)chiral superfields we should take into account the following identity,

$$\left(D_\alpha \Phi^+\right) = 0, \quad \left(\bar{D}_{\dot{\alpha}} \Phi\right) = 0 \tag{35}$$

So when D^2 acts on two superfields, if there is only one Φ, all of the D will act on Φ,

$$\left(D^2\Phi\right)\Phi^+ \tag{36}$$

if there are two Φ's, the possible forms are as follows,

$$\left(D_\alpha\Phi\right)\left(D_\beta\Phi\right), \quad \left(D^2\Phi\right)\Phi \tag{37}$$

When there is $\partial D\bar{D}$ in the operators, we can also determine the distribution by using eq. (35), that is, D acting on Φ, $\bar{D}$ acting on Φ^+. If not, let us try to act them on one superfield,

$$\Phi\left(\partial D\bar{D}\Phi^+\right) \tag{38}$$

then by using D algebraic relation we have

$$\Phi\left(\Box\Phi^{+}\right) \tag{39}$$

This contradicts with our initial postulate that there is $\partial D\bar{D}$ in the operators.

5.2.3 Fierz Identity

Let us consider the following action form,

$$\int d^4x\left(\partial_k\partial_l D_\alpha\Phi\right)\left(D_\beta\Phi\right) \tag{40}$$

we can transform eq. (40) as follows,

$$-\int d^4x\left(\partial_k\partial_l D_\beta\Phi\right)\left(D_\alpha\Phi\right) \tag{41}$$

by using partial derivatives identity eq. (34) twice, where we have included -1 by exchanging Fermionic number. Then by using Fierz identity,

$$A_\alpha B_\beta - A_\beta B_\alpha = -\epsilon_{\alpha\beta}\epsilon^{\gamma\zeta}A_\gamma B_\zeta \tag{42}$$

we can transform eq. (40) to the following form,

$$\frac{-1}{2}\epsilon_{\alpha\beta}\epsilon^{\gamma\zeta}\int d^4x\left(\partial_k\partial_l D_\gamma\Phi\right)\left(D_\zeta\Phi\right) \tag{43}$$

which is antisymmetry with respect to α,β. Eq. (43) can only combine with constant form which has no symmetry with respect to α,β.

5.2.4 Constraint by $1/2$ Supersymmetry

For the operators $\Box\Phi\Phi^{+}$, we find there are only the following forms in $S_{(3)}$,

$$\Lambda^2\eta^{kl}, \quad \sigma\Lambda\Lambda\eta^{kl} \tag{44}$$

which can combine with $\partial_k\partial_l$. According to eq. (27), there should contain the following form,

$$\left(\eta\sigma\Lambda\Lambda^k\right)^l, \quad \Lambda^{kl}, \tag{45}$$

Further more, we note that in $S_{(3)}$,

$$\partial D\overline{D}\Phi\Phi^{+}, \quad \Box\Phi\Phi^{+}, \quad D^2\overline{D}^2\Phi\Phi^{+} \tag{46}$$

form a 1/2 supersymmetry invariant subset. The indices of $\partial D\overline{D}\Phi\Phi^+$ can also combine with,

$$\Lambda^{kl}\eta_{ln}\left(\bar{\sigma}^n\right)^{\dot{\alpha}\beta}, \quad \eta_{nl}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta}\left(\eta\sigma\Lambda\Lambda^n\right)^k,$$
$$\eta_{nl}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^n \tag{47}$$

there are not such forms in the action, either.

In order to explain this phenomenon, we combine all of the possible constant forms with operators in eq. (46) to construct 1/2 supersymmetry invariant subsets. Then we make 1/2 supersymmetry transformation on them, set the result be 0. We see that we can not ensure the 1/2 supersymmetry invariance if we add new combination of constant form and operators. By this way, we understand why there are not such combination in $S_{(3)}$. In other words, we not only have to match the indices, but also constraints of 1/2 supersymmetry invariance.

In conclusion, by using eq. (26), (27), (28), (29), (30), (31), and (32) and taking into account the constant forms,

$$\epsilon^{\alpha\zeta}\eta_{no}\left(\sigma\Lambda^{ok}\right)^{l\beta}\Lambda^n{}_\zeta, \quad \epsilon^{\alpha\zeta}\left(\eta\sigma\Lambda^k\right)^\beta\Lambda^l{}_\zeta,$$
$$\eta^{kl}\epsilon^{\alpha\zeta}\eta_{no}\left(\eta\sigma\Lambda^n\right)^\beta\Lambda^o{}_\zeta \tag{48}$$

we transform them to the following forms,

$$\left(\eta\sigma\Lambda^k\right)^l, \sigma\Lambda\Lambda\eta^{kl} \tag{49}$$

When they combine with $\epsilon_{\alpha\beta}$, we can determine all of the action corresponding to the 2 point operator. By comparing with $S_{(3)}$, we find the following forms are not contained in $S_{(3)}$,

$$\Lambda^{kl}\theta^4\Phi^+\partial_k\partial_l\Phi^+$$
$$\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\Phi^+\partial_k\partial_l\Phi^+ \tag{50}$$

By applying the same consideration for 3, 4, 5, 6 point operators, we can construct all possible actions for the operators in the fourth class and find that most of them are contained in $S_{(3)}$, except the following forms,

$$\Lambda^{kl}\theta^4\Phi^+\Phi^+\partial_k\partial_l\Phi^+$$
$$\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\Phi^+\Phi^+\partial_k\partial_l\Phi^+ \tag{51}$$

6 Form in Higher Order Loop Expansion

We see that the forms in eqs.(18), (50), and (51) are not contained in $S_{(3)}$. In the above analysis we did not use the explicit form of $S_{(3)}$. By using action $S_{(3)}$, we observe the vertices that can produce forms in eqs. (18), (50), and (51). We note that for Λ^2 order, all of the loop expansion contain only one BFNC vertex. Some of the forms in eq. (18) are produced at 3 loop expansion. We note that we have found many graphs, to analyze all of them is very complex and beyond the scope of this work. We just need to illustrate that the forms in eqs.(18), (50), and (51) are indeed produced at higher loop expansion by using several examples. We will neglect the constant part of the term in the action $S_{(3)}$ and only present their superfield parts.

The following term,

$$\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta} \theta^4 \left(D_\beta \Phi\right) \partial_k \left(D^2 \Phi\right) \left(\bar{D}_{\dot{\alpha}} \Phi^+\right) \tag{52}$$

will produce $(\Phi^+)^n$, $n = 1, 2, 3, 4$ as illustrated in Figure 1.

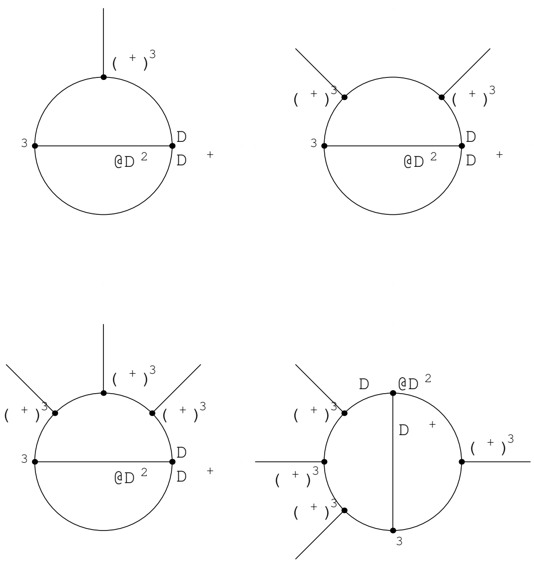

Figure 1: These will produce $(\Phi^+)^n$, $n = 1, 2, 3, 4$.

The following term,

$$\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta} \theta^4 \left(D_\beta \Phi\right) \left(D^2 \Phi\right) \Phi^+ \partial_k \left(\bar{D}_{\dot{\alpha}} \Phi^+\right) \tag{53}$$

will produce $(\Phi^+)^n$, $n = 5$ as illustrated in Figure 2.

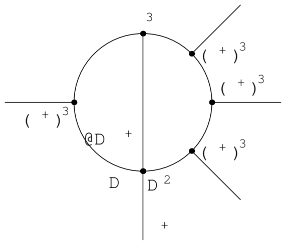

Figure 2: This will produce $(\Phi^+)^n$, $n = 5$.

The following term,

$$\theta^4 \left(D^2\Phi\right) \left(D^2\Phi\right) \left(\bar{D}^2\Phi^+\right) \tag{54}$$

will produce $\left(\bar{D}^2\Phi^+\right) (\Phi^+)^n$, $n = 0, 1, 2, 3, 4$ as illustrated in Figure 3.

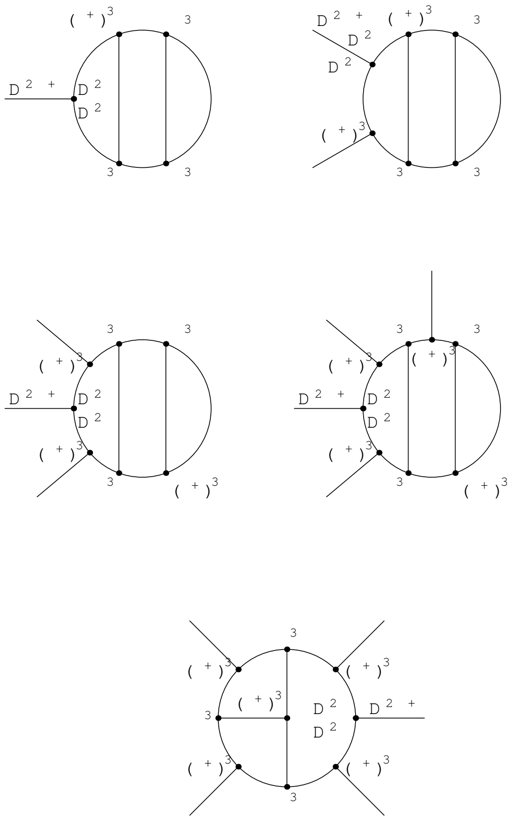

Figure 3: These will produce $\left(\bar{D}^2\Phi^+\right) (\Phi^+)^n$, $n = 0, 1, 2, 3, 4$.

The following term,

$$\theta^4 \left(D^2\Phi\right) \left(D^2\Phi\right) \Phi^+ \left(\bar{D}^2\Phi^+\right) \tag{55}$$

will produce $\left(\bar{D}^2\Phi^+\right) (\Phi^+)^n$, $n = 5$ as illustrated in Figure 4.

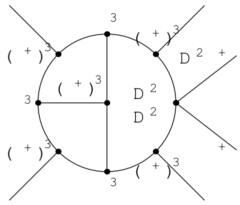

Figure 4: This will produce $\left(\bar{D}^2\Phi^+\right)\left(\Phi^+\right)^n$, $n=5$.

The following term,

$$\Lambda^{kl}\theta^4\Phi\left(D^2\Phi\right)\Phi^+\partial_k\partial_l\Phi^+ \tag{56}$$

will produce

$$\begin{aligned}&\Lambda^{kl}\Phi^+\partial_k\partial_l\Phi^+\\ &\Lambda^{kl}\Phi^+\Phi^+\partial_k\partial_l\Phi^+\end{aligned} \tag{57}$$

The following term,

$$\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\Phi\left(D^2\Phi\right)\Phi^+\partial_k\partial_l\Phi^+ \tag{58}$$

will produce

$$\begin{aligned}&\left(\eta\sigma\Lambda\Lambda^k\right)^l\Phi^+\partial_k\partial_l\Phi^+\\ &\left(\eta\sigma\Lambda\Lambda^k\right)^l\Phi^+\Phi^+\partial_k\partial_l\Phi^+\end{aligned} \tag{59}$$

The contributions of eqs. (56) and (58) are illustrated in Figure 5.

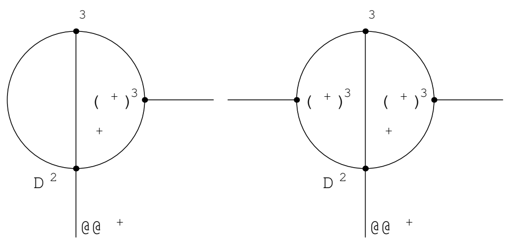

Figure 5: The left will produce $\Lambda^{kl}\Phi^+\partial_k\partial_l\Phi^+$ and $\left(\eta\sigma\Lambda\Lambda^k\right)^l\Phi^+\partial_k\partial_l\Phi^+$.
The right will produce $\Lambda^{kl}\Phi^+\Phi^+\partial_k\partial_l\Phi^+$ and $\left(\eta\sigma\Lambda\Lambda^k\right)^l\Phi^+\Phi^+\partial_k\partial_l\Phi^+$

7 Conclusion and Outlook

In ref. [12] we start from the Wess-Zumino model and use m and g as mass and interacting parameters. By using iteration method, we calculate effective action for several times. By introducing the definition of 1/2 supersymmetry invariant subsets and their bases, we construct one-loop renormalizable action $S_{(3)}$.

In order to study the renormalization to all order loops by using the symmetry analysis method, we introduce m, g, m^*, g^* as mass and interacting parameters in eq. (1) and define the BFNC Wess-Zumino model by using BFNC $\star$-product.

By using dimensional analysis and charge considerations, we obtain all possible operators in eqs. (16), (17), (18), (19), (20), (21), (22), (23), and (24). We try to construct all possible forms of action by using them.

We also find that we can redefine the parameters $x_{i,j}, y_{i,j}, z_{i,j}$ of B_i and denote as B'_i, which has a proper mass dimension and satisfies the $U(1)_R$ R-symmetry and $U(1)_\Phi$ flavor symmetry.

By comparison of the action constructed from operators and terms in B'_i, we obtain all but a few are contained in B'_i. The terms not contained in $B'{}_i$ can be produced by the vertices of B'_i at higher order loop expansion.

We construct action $S'_{(3)}$,

$$S'_{(3)} = S_{WZ} + \sum_{i=1}^{74} B'_i \tag{60}$$

and deduce that $S'_{(3)}$ plus the form in eqs.(18), (50), and (51) are the all order renormalizable Wess-Zumino action on the BFNC superspace.

Our above analysis is made at the second order of BFNC parameters. For obtaining a renormalizable action at higher order of BFNC parameters, we can firstly do symmetry analysis and obtain all operator forms, then construct all possible actions from them. At the end we construct 1/2 supersymmetry invariant subsets and their corresponding bases. The result will be all order renormalizable.

Acknowledgments

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